Contents

- Wojtek Dorabialski:
  - Choice Under Uncertainty
  - Game Theory
- Olga Kiuila
  - Advanced Consumer & Producer Theory
  - General Equilibrium Theory

How to complete successfuly

- Attend classes
- Read lecture notes at: www.wne.uw.edu.pl/kiuila/am
- Read textbooks
- Solve problem sets
- Ask for help (office hours)
- Prepare for and pass both parts of the final exam

Expected Utility Theory

- Toolkit for analyzing choice under uncertainty
- There is a finite set C of possible outcomes, indexed by $n = 1, ..., N$.
- Probabilities of various outcomes are **objectively known**
- A **simple lottery (L)** is a list of probabilities of each outcome: $L = (p_1,..., p_N)$, $p_n \geq 0$ for all n and

  $p_1 + ... + p_N = 1$

Compound lotteries

- Given k simple lotteries, L_k , and probabilities $\alpha_k \geq 0$ with
$\alpha_1 + ... + \alpha_N = 1$, the compound lottery $(L_1, ..., L_k; \alpha_1, ..., \alpha_k)$ is a risky alternative (lottery) which yields the simple lottery L_k with probability α_k for $k = 1, ..., K$
- For any compound lottery, we can calculate a corresponding **reduced lottery**, which generates the same distribution of outcomes,

  $L_R = \alpha_1 L_1 + ... + \alpha_K L_K$
- The probabilities in the reduced lottery can be calculated as follows:

  $p_n = \alpha_1 p_n^1 + ... + \alpha_K p_n^k = 1$

  Example: What is the reduced lottery?

  CL: (1/3, 1/3, 1/3) over (1, 0, 0); (1/4, 3/8, 3/8); (1/4, 3/8, 3/8)

  CL: (1/2 1/2) over (1/2, 1/2, 0); (1/2, 0, 1/2)

$\triangleright L, L', L'' \in \Lambda$

Preferences over lotteries

- Continuity Assumption:
  - The preference relation $\triangleright$ on the space of simple lotteries Λ is *continuous* if for any

$L, L', L'' \in \Lambda$ the sets

$\triangleright$

$$\{\alpha \in [0,1] : \alpha L + (1-\alpha)L' \triangleright L''\} \subset [0,1]$$

  - and

$$\{\alpha \in [0,1] : \alpha L + (1-\alpha)L' \triangleleft L''\} \subset [0,1]$$

  - are closed
  - This excludes lexicographic preferences over outcomes

Preferences

- Independence Assumption:
  - The preference relation $\triangleright$ on the space of simple lotteries Λ satisfies *independence* if for any
  - $L, L', L'' \in \Lambda$ we have $L \triangleright L'$ iff

$$\alpha L + (1-\alpha) L'' \triangleright \alpha L' + (1-\alpha) L''$$

  - This makes perfect sense: a preference between two lotteries should not depend on any alternative outcome (or lottery) that may occur instead

Expected Utility

- A utility function, which represents preferences over lotteries, has an *expected utility (von Neumann-Morgenstern) form* if there is an assignment of numbers ($u_1, \ldots, u_N$) to the outcomes such that for every simple lottery L

$$U(L) = u_1 p_1 + \ldots + u_N p_N.$$

- Properties of v.N-M utility function
  - linearity: $$U\left(\sum_{k=1}^{K} \alpha_k L_k\right) = \sum_{k=1}^{K} \alpha_k U(L_k)$$
  - invariance to affine transformations, the function:

$$\tilde{U}(L) = \beta U(L) + \gamma$$

represents the same preferences as the function $U(L)$, as long as $\beta > 0$ and γ are scalars

Expected Utility Theorem

- If preferences satisfy continuity and independence, then they can be represented by a v.N-M utility function.
- Graphical explanation:
  - v.N-M representation means that indifference curves are straight and parallel lines
  - If independence is violated, indifference curves will not be straight or will not be parallel

Pros and Cons of Expected Utility

- Pros:
  - convenient, difficult to do without
  - people who have v.N-M preferences can use extrapolation to assess risky alternatives
- Cons:
  - Allais paradox
  - Machina's paradox

An experiment

- Which monetary lottery would you choose?
- A

2 500 000	500 000	0
0	1	0

- or B

2 500 000	500 000	0
0.10	0.89	0.01

An experiment cntd

- Which monetary lottery would you choose?
- C

2 500 000	500 000	0
0	0.11	0.89

- or D

2 500 000	500 000	0
0.10	0	0.90

Allais Paradox

- If you have chosen A and D, your preferences do not satisfy independence (cannot be represented by a v.N-M utility function)
- Proof: the choice of A over B implies

$$u_{0.5} > (0.1)u_{25} + (0.89)u_{0.5} + (0.01)u_0$$

adding $(0.89)u_0$ - $(0.89)u_{0.5}$ to both sides yields

$$(0.11)u_{0.5} + (0.89)u_0 > (0.1)u_{25} + (0.90)u_0$$

hence an individual who also chose D over C cannot have a v.N-M utility function

Reactions to Allais paradox

- Ignore it, people are rarely faced with such extreme choices
- Regret theory: the possibility of getting zero instead of an assured nice outcome scares people away (they want to avoid regret). There isn’t such threat in the second choice.
- Replace independence assumption with something weaker

Machina's Paradox

- Suppose there are 3 outcomes:
  - A: a trip to Venice
  - B: seeing an excellent movie about Venice in the cinema
  - C: staying at home
- Typically people prefer A to B and B to C
- Which lottery would you choose?

A	B	C
0.99	0.01	0

or

A	B	C
0.99	0	0.01

explanation: disappointment aversion